F I R Filter
(Finite Impulse Response)

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The Outline

State of the art

- The basic FIR filter is characterized by the follo-wing two equations :

$$y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)$$

$$H(z) = \sum^{N-1} h(k)$$

- FIR filter can have an exactly linear phase response

State of the art (cont'd)

- One of the most important properties of FIR filters is the ability to have an exactly linear phase res-ponse
- The phase delay or group delay of the filter pro-vides a useful measure of how the filter modifies the phase characteristic of the signal

$$T_p = -\frac{\theta(\omega)}{\omega} \qquad T_g = -\frac{d\theta(\omega)}{d\omega}$$

PHASE DELAY

GROUP DELAY

State of the art (cont'd)

- The distortion in many applications (e.g. in music, video, biomedicine) is undesirable signal. It can be avoided by using filters with linear phase characteristic over the frequency band of interest
- The filter having a linear phase response, satisfies one of the following relationships.

$$\theta(\omega) = -\alpha\omega$$

$$\theta(\omega) = \beta - \alpha\omega$$

→ α and β are constant

Type of Linear Phase Filters

- There are 4 types of linear phase FIR filters
- It depends on whether *N* is even or odd and whether $h(n)$ has positive or negative symmetry

$$\theta(\omega) = -\alpha\omega$$ → Positive symmetry

$$\theta(\omega) = \beta - \alpha\omega$$ → Negative symmetry

Summary of 4 types FIR Filters

Table 7.1

Impulse response symmetry	*Number of coefficients N*	*Frequency response $H(\omega)$*	*Type of linear phase*
Positive symmetry, $h(n) = h(N-1-n)$	Odd	$e^{-j\omega(N-1)/2} \sum_{n=0}^{(N-1)/2} a(n)\cos(\omega n)$	1
	Even	$e^{-j\omega(N-1)/2} \sum_{n=1}^{N/2} b(n)\cos[\omega(n-\frac{1}{2})]$	2
Negative symmetry, $h(n) = -h(N-1-n)$	Odd	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=1}^{(N-1)/2} a(n)\sin(\omega n)$	3
	Even	$e^{-j[\omega(N-1)/2-\pi/2]} \sum_{n=1}^{N/2} b(n)\sin[\omega(n-\frac{1}{2})]$	4

$a(0) = h[(N-1)/2]$; $a(n) = 2h[(N-1)/2 - n]$
$b(n) = 2h(N/2 - n)$

Comparison of the impulse of the 4 types FIR filters

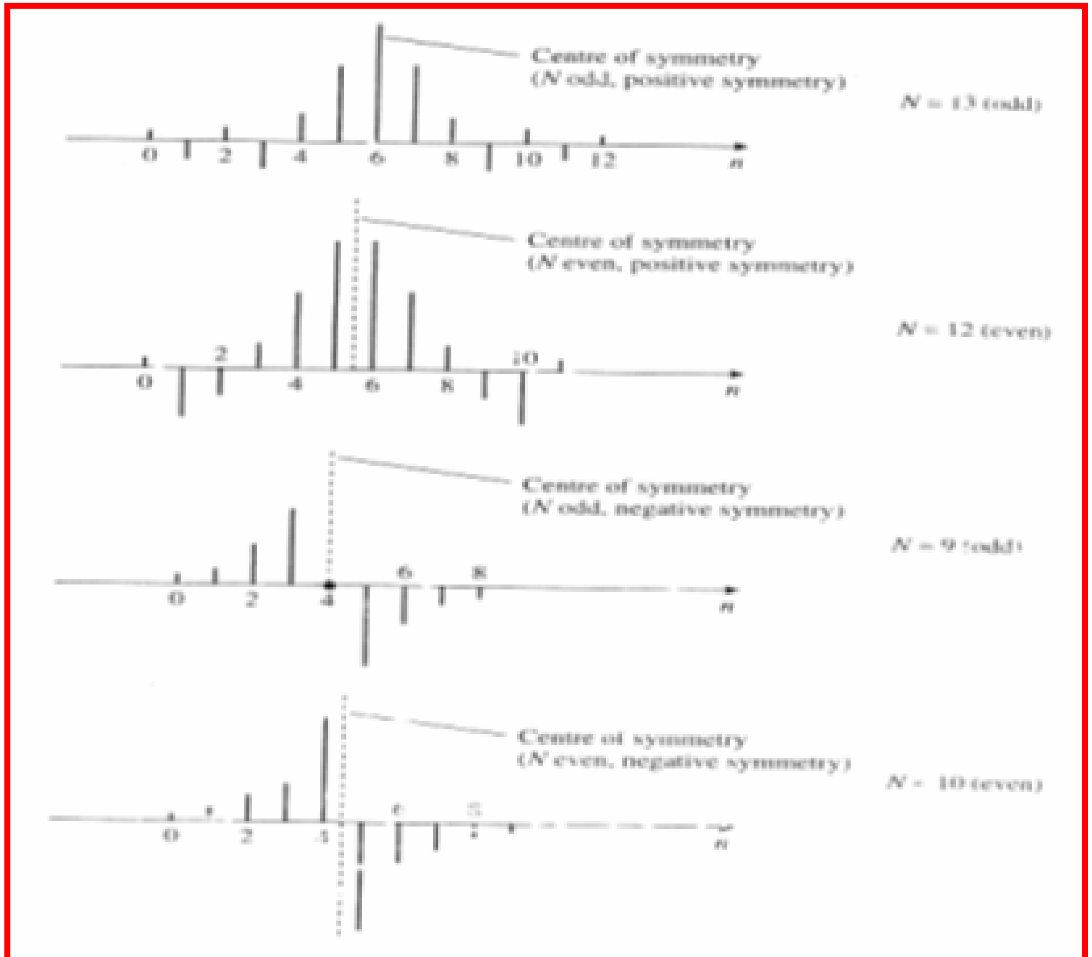


Figure 7.1

FIR Coefficient calculation method

- There are some methods to calculate FIR coefficient but only 3 are the most commonly used
- FIR coefficient calculation has an objective to obtain values of $h(n)$ such that resulting filter meets the design specifications, amplitude-frequency response and throughput requirements
- Remember again the two equations that show FIR characteristic ! (in 1^{st} page)

Window Method

- The main key is : the frequency response of a filter, $\boldsymbol{H_D(\omega)}$, and the corresponding impulse response, $\boldsymbol{h_D(n)}$, are related by the inverse Fourier transform :

$$h_D(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_D(\omega) e^{j\omega n} d\omega$$

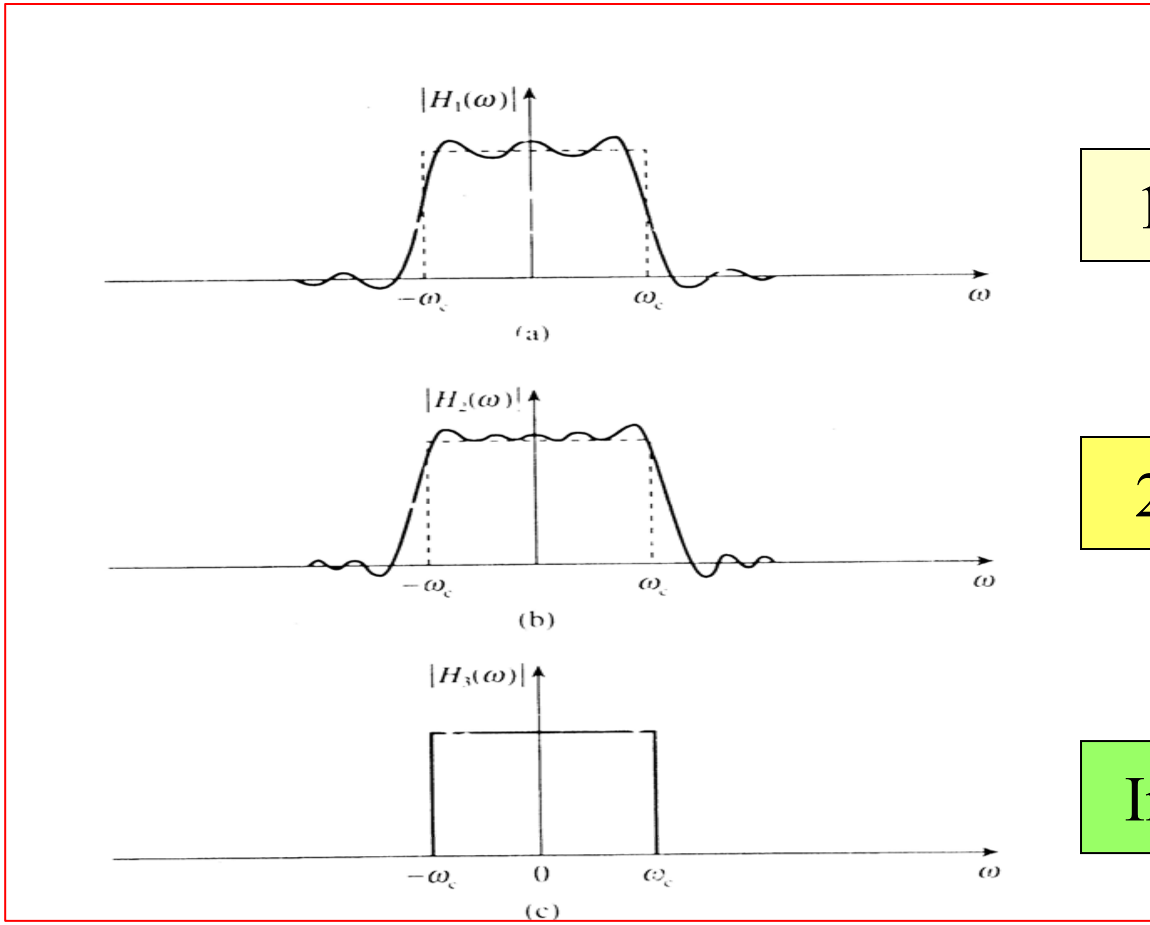
Note that the subscript D means ideal function

Window Method (cont'd)

- The subscript D is used to distinguish between the ideal and practical impulse responses
- A practical approach is to multiply the ideal impulse response, $\boldsymbol{h_D(n)}$ by a suitable **window function** $\boldsymbol{w(n)}$, whose duration is finite
- So that, the resulting impulse response decays smoothly towards zero

Look at the figure 7.2 below
Effects on the frequency response

Effects on the frequency response of truncating the ideal impulse response to 13, 25, and infinite number coefficients



13 coefficients

25 coefficients

Infinite number

Figure 7.2

Window Method (cont’d)

- Here are 4 steps to obtain FIR coefficients by Window Method

STEP 1 : Specify the ideal frequency response $\boldsymbol{H_D(\omega)}$

STEP 2 : Obtain the impulse response $\boldsymbol{h_D(n)}$ by using Inverse Fourier Transform

STEP 3 : Select a window function $\boldsymbol{w(n)}$

STEP 4 : Obtain values of actual FIR coefficients $\boldsymbol{h(n)}$ by multiplying $\boldsymbol{h_D(n)}$ and $\boldsymbol{w(n)}$

Window Method (cont’d)

- Look at Figure 7.3 on next slide. It is an illustra-tion of **the process** how the filter coefficients, $h(n)$ are determined by the window method
- Figure 1a shows the **ideal frequency response and the corresponding ideal impulse response**
- Figure 1b shows the **finite duration window function and its spectrum**
- Figure 1c shows ***h*(*n*) which is obtained by multiplying figure 1a and 1b.**

The Window Method Illustration

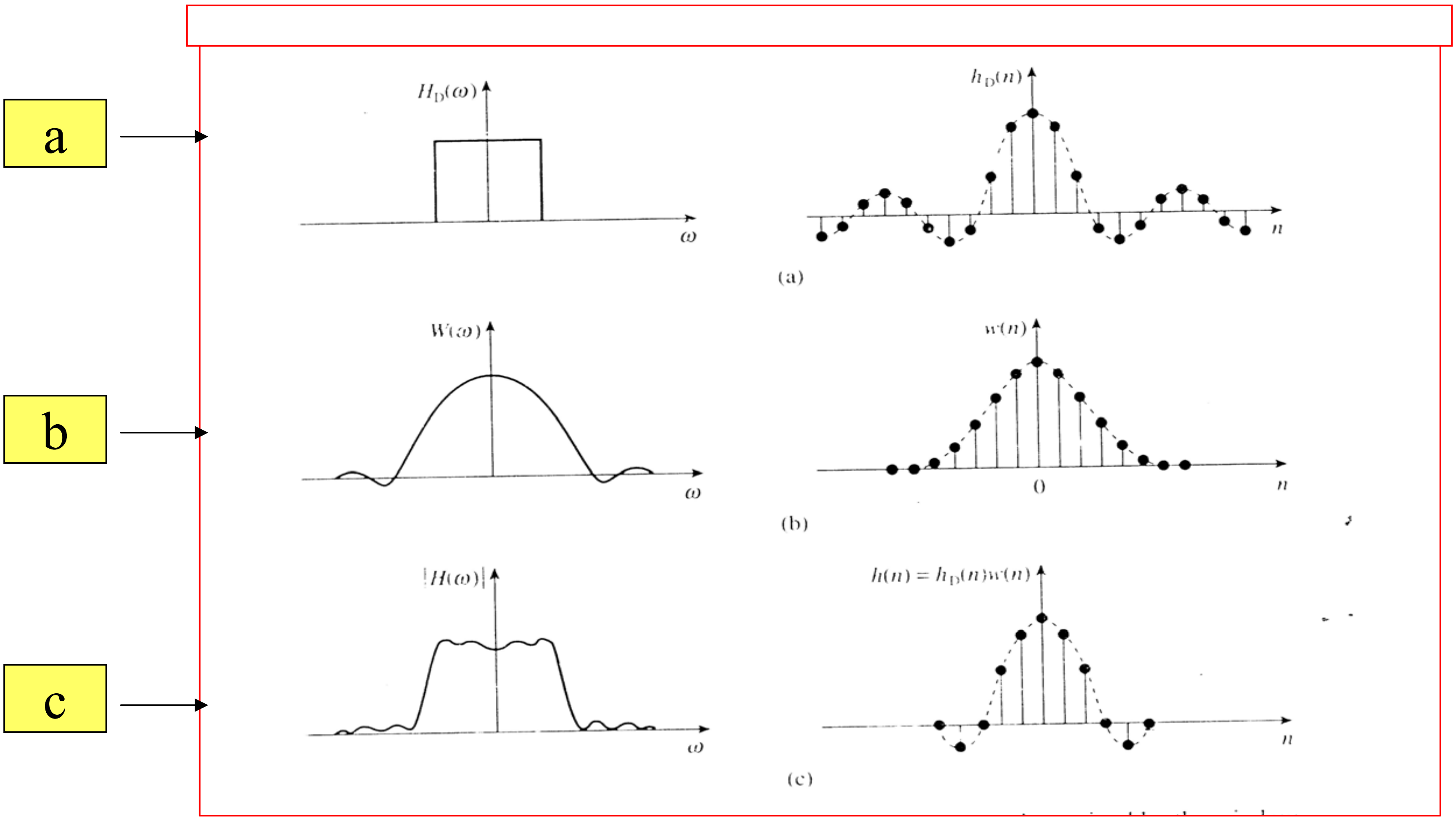


Figure 7.3

Window Method (cont'd)

- There are 3 common window method functions
  Rectangular, *Hamming*, and *Blackman*
- The parameters for choosing the method are summarized on the table → next 4 pages
- Note the parameters :
  - transition width (Hz)
  - passband ripple and stopband attenuation (dB)
  - mainlobe relative to side lobe (dB)

Window Method (cont'd)

- Here is the comparison of the time and frequency domain characteristics of ***Rectangular*** *Window functions*

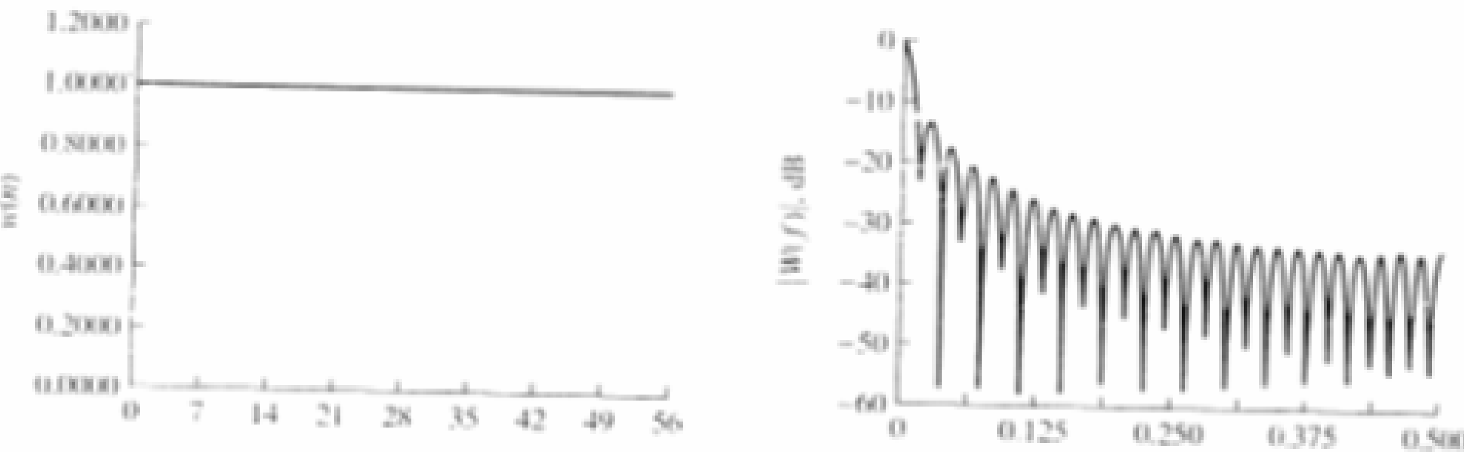


Figure 7.4

Window Method (cont'd)

- Here is the comparison of the time and frequency domain characteristics of ***Hamming*** *Window functions*

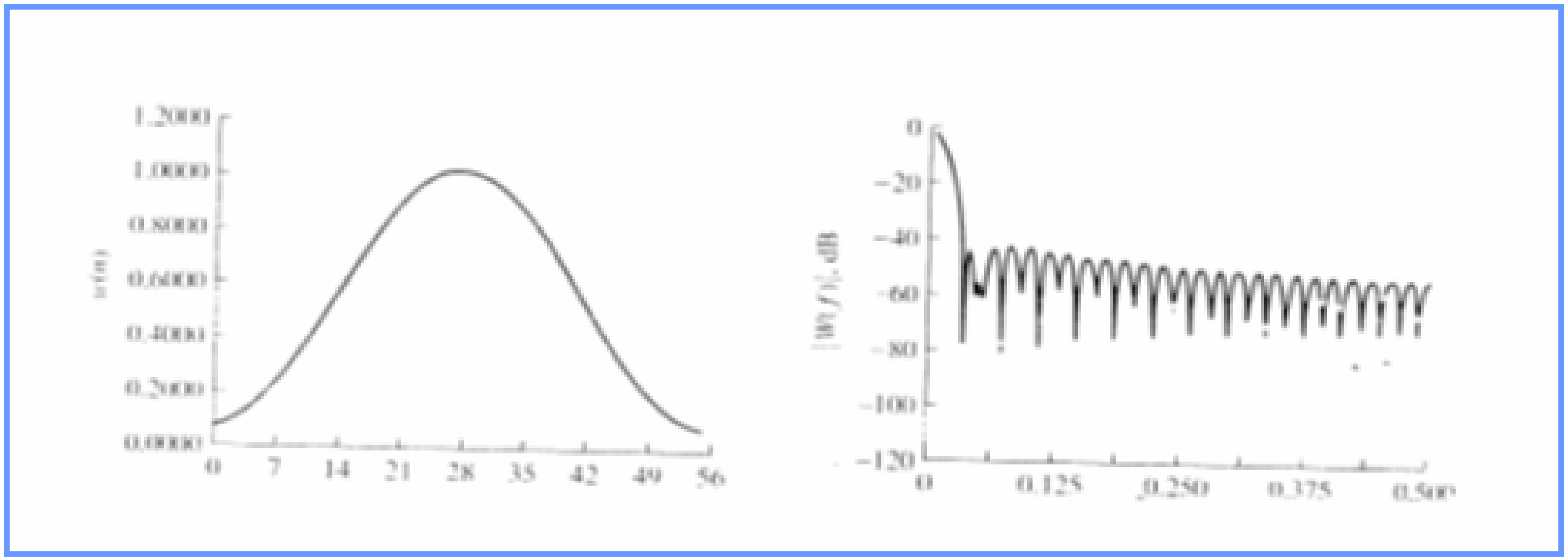


Figure 7.5

Window Method (cont’d)

- Here is the comparison of the time and frequency domain characteristics of ***Blackman*** *Window functions*

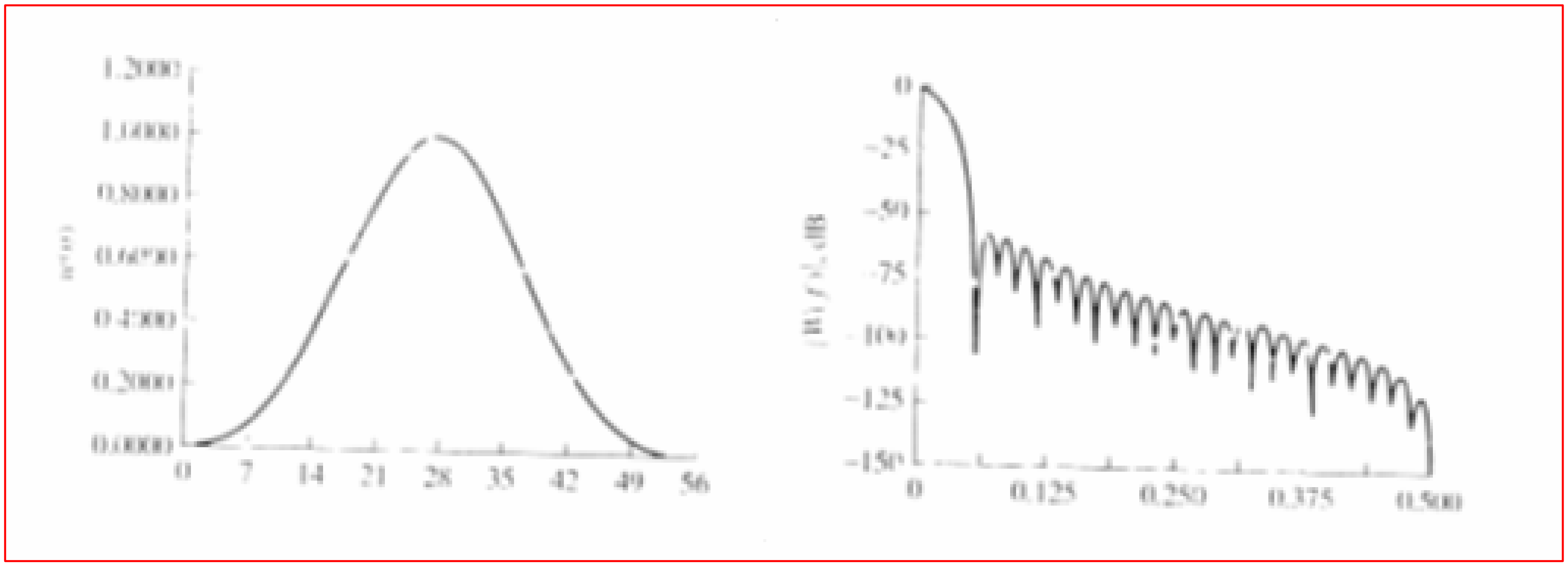


Figure 7.6

Table of Window Method Parameters

Table 7.2

Name of window function	Transition width (Hz) (normalized)	Passband ripple (dB)	Main lobe relative to side lobe (dB)	Stopband attenuation (dB) (maximum)	Window function $w(n), \|n\| \leq (N-1)/2$
Rectangular	$0.9/N$	0.7416	13	21	1
Hanning	$3.1/N$	0.0546	31	44	$0.5 + 0.5\cos\left(\frac{2\pi n}{N}\right)$
Hamming	$3.3/N$	0.0194	41	53	$0.54 + 0.46\cos\left(\frac{2\pi n}{N}\right)$
Blackman	$5.5/N$	0.0017	57	75	$0.42 + 0.5\cos\left(\frac{2\pi n}{N-1}\right) + 0.08\cos\left(\frac{4\pi n}{N-1}\right)$
	$2.93/N\ (\beta = 4.54)$	0.0274		50	$\frac{I_0(\beta\{1 - [2n/(N-1)]^2\}^{1/2})}{I_0(\beta)}$
Kaiser	$4.32/N\ (\beta = 6.76)$	0.002 75		70	
	$5.71/N\ (\beta = 8.96)$	0.000 275		90	

The Optimal Method

- This method is very powerful, flexible and very easy to apply
- This method is based on the concept of equiripple passband and stopband
- Remember these following parameters :

1. **N** : the number of filter coefficients, that is filter length. The value of N, can be obtained by some pattern, e.g. lowpass, bandpass
2. **Jtype** : the type of filter, e.g. multiple passband or stopband filters on lowpass, highpass.

The Optimal Method (cont'd)

3. ***W(ω)*** : the weighting function. This specifies the relative importance of each band
4. **Ngrid** : grid density. This is the number of frequency points at which, during the process of finding the external frequencies. The default for Ngrid value is 16.
5. **Edge** : the bandedge frequencies, the lower and upper bandedge freq. for filter. All value must be entered in normalized form.

The Optimal Method (cont'd)

- Here are the 7 steps to obtain filter coefficient :

STEP 1 : Specify the bandedge frequencies, passband ripple and stopband attenuation and sampling freq.

STEP 2 : Normalize each bandedge frequency by dividing it by the sampling frequency

STEP 3 : Use the passband ripple and stopband attenuation expressed in ordinary units, to estimate $\boldsymbol{N}$

The Optimal Method (cont'd)

STEP 4 : Obtain the weights for each band from the ratio of the passband to stopband ripples, expressed in ordinary units

STEP 5 : Input the parameters to the optimal design program to obtain the coefficients: *N* bandedge frequencies and weights, grid density

STEP 6 : Check the passband ripple and stopband attenuation produced by the program

The Optimal Method (cont'd)

STEP 7 : If the specifications are not satisfied, increase the value of *N* and repeat steps 5 and 6 until they are; then obtain and check the frequency response to ensure that it satisfies the specifications

↓

USE THE COMPUTER PROGRAM

7.5.3

Frequency Sampling Method

Refer to your text books

Make resume about this topic

FIR implementation

- Here is a simplified block diagram of real-time digital filter with analog input/output signals

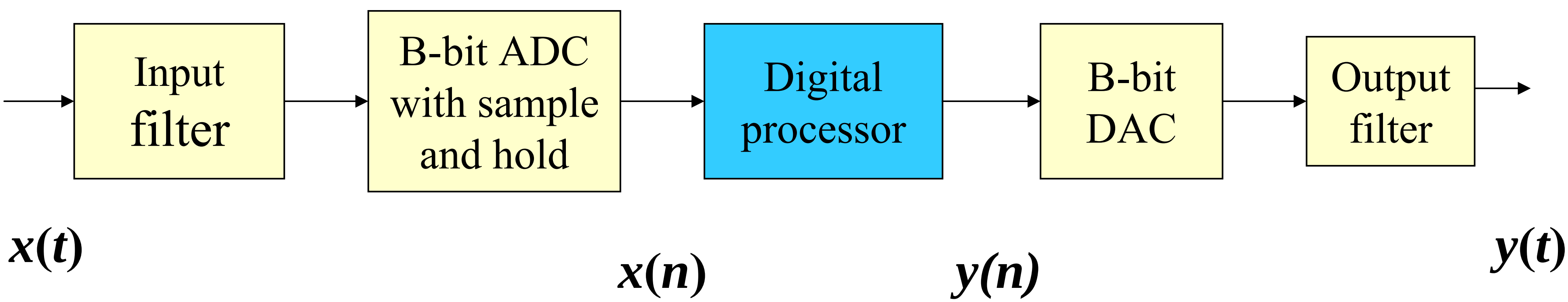


Figure 7.7